Resonance in the seesaw mechanism

Haijun Pan[1*] and G. Cheng[2,3†]

[1] *Lab of Quantum Communication and Quantum Computation, and Center of Nonlinear Science, University of Science and Technology of China, Heifei, Anhui, 230026, P.R.China*

[2] *CCAST(World Laboratory), P.O.Box 8730, Beijing 10080, P.R.China*

[3] *Department of Astronomy and Applied Physics, University of Science and Technology of China, Heifei, Anhui, 230026, P.R.China*

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Abstract

We study the RH neutrino properties from the low energy neutrino data in the seesaw mechanism. Reonance behavior is found for the right-handed (RH) mixing angle as a function of light neutrino mass ratios in two favored region of the solar neutrino problem and, at the resonance point, the two corresponding RH Majorana neutrino masses are degenerate. This phenomenon is similar with that in the matter-enhanced conversion of neutrinos. The physical significance it infers for the electron neutrino mass and other neutrino parameters is discussed.

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*Email: phj@mail.ustc.edu.cn

†Email: gcheng@ustc.edu.cn

The smallness of left-handed (LH) neutrino masses implied by the solar and atmospheric neutrino experiments [1,2] is perhaps attributed to the Majorana feature of neutrino fields [3,4]. By adding the right-handed (RH) fields ν_R of heavy neutrinos, the seesaw mechanism [5] provides a very natural and attractive explanation of the smallness of the neutrino masses compared to the masses of the charged fermions. In this mechanism the LH Majorana neutrino mass matrix, m_ν, is expressed in the following form [6]:

$$m_\nu = m_D M^{-1} m_D^T, \tag{1}$$

where m_D is the Dirac mass matrix which, as suggested by Grand Unified Theories (GUTs), is assumed to be similar to that in the quark sector and M is the Majorana mass matrix for the RH neutrino components. After the pioneer work of Smirnov, interest in determining the properties of the RH Majorana neutrinos has been revised with expanding experimental data about neutrinos (see [7–9]).

The Dirac matrix can be diagonalized by the bi-unitary transformation [4]: $m_D^{\rm diag} = D_L^\dagger m_D D_R = {\rm diag}\{m_{1D}, m_{2D}, m_{3D}\}$. m_ν and M are in general complex matrices and, respectively, only a single unitary matrix is needed for the diagonalization: $m_\nu^{\rm diag} = U^T m_\nu U = {\rm diag}\{m_1, m_2, m_3\}$, $M^{\rm diag} = V_0^T M V_0 = {\rm diag}\{M_1, M_2, M_3\}$. Eq. (1) can be rewritten as

$$V (M^{\rm diag})^{-1} V^T = (m_D^{\rm diag}) (S^\dagger)^T m_\nu^{\rm diag} S^\dagger (m_D^{\rm diag}) \equiv X, \tag{2}$$

where $V = D_R^\dagger R$ is the RH mixing matrix containing the contributions from the diagonalizations of both m_D and M^{-1} and can be parameterized by $V = e^{i\beta_{23}\lambda_7} e^{i\beta_{13}\lambda_5} e^{i\beta_{12}\lambda_2}$ with $\lambda_{2,5,7}$ the Gell-Mann matrices (for example, see [10]). $S = D_L^T U$ is so-called seesaw matrix which specifies the feature of the seesaw mechanism [6]. In this letter, assuming quark-lepton symmetry and hierarchical Dirac and LH Majorana neutrino spectra, we aim at analyzing the RH neutrino properties (masses and mixing) from the low energy neutrino data. The CP-violating effect will be ignored in our analysis and so all the transformation matrices entered the seesaw mechanism will be real orthogonal. Due to the large difference among different eigenvalues of X (usually they can be apart away more than about 10 magnitudes), it is hard to obtain them precisely even numerically. Even this is done, the complicated expressions of them make it impossible to see explicit dependence on various physical parameters. We notice that the left side of Eq. (2) contains the contributions mainly from M_1^{-1} and M_2^{-1} while its inverse $V(M^{\rm diag})V^T$ contains the contributions mainly from M_2 and M_3. Two of the eigenvalues can be obtained through solving a quadratic equation and the third is then obtained by the equation of the determinants of both sides of Eq. (2) [9].

As suggested by Smirnov, D_L is nearly a unit matrix and then $U \approx S$ which can be parameterized as

$$U = e^{i\theta_{23}^{\nu}\lambda_7} e^{i\theta_{13}^{\nu}\lambda_5} e^{i\theta_{12}^{\nu}\lambda_2}. \tag{3}$$

Among the three angles, θ_{13}^{ν} is small implied by the CHOOZ observation [11], θ_{23}^{ν} is almost maximal (i.e. $\theta_{23}^{\nu} \approx \frac{\pi}{4}$) which is suggested strongly by the recent result from SuperKamiokande [2] while θ_{12}^{ν} which is responsible for the solar neutrino deficit can vary from rather small value (10^{-3}) (for SMA: small angle MSW [12] effect) to nearly $\pi/4$ (for VO: vacuum oscillation, LAM: large angle mixing MSW effect and LOW: low mass or possibility) [13].

When θ_{12}^{ν} is small, we consider a special case: when $\theta_{12}^{\nu} \sim \theta_{13}^{\nu}$. For convenient, we set $U_{\tau 1} = 0$. The three RH Majorana masses are given by [14]

$$M_1 \approx \frac{m_{1D}^2}{m_3} \cot^2 \theta_{12}^{\nu}, \tag{4a}$$

$$M_2 \approx \begin{cases} 2\frac{m_{2D}^2}{m_1} \sin^2 \theta_{12}^{\nu}, & \text{if} \quad r_{21} < r_{21}^{\text{res}} \\ \frac{1}{2}\frac{m_{3D}^2}{m_2}, & \text{if} \quad r_{21} > r_{21}^{\text{res}} \end{cases} \tag{4b}$$

$$M_3 \approx \begin{cases} \frac{1}{2}\frac{m_{3D}^2}{m_2}, & \text{if} \quad r_{21} < r_{21}^{\text{res}} \\ 2\frac{m_{2D}^2}{m_1} \sin^2 \theta_{12}^{\nu}, & \text{if} \quad r_{21} > r_{21}^{\text{res}} \end{cases} \tag{4c}$$

where $r_{21} = \frac{m_2}{m_1}$ and $r_{21}^{\text{res}} = \frac{1}{4}\frac{m_{3D}^2}{m_{2D}^2} \csc^2 \theta_{12}^{\nu}$. Note that we have two degenerate masses $M_2 = M_3$ when $r_{21} = r_{21}^{\text{res}}$.

The second RH mixing angle β_{23} is given in

$$\sin 2\beta_{23} \approx -\frac{2m_{2D}/m_{3D}}{\sqrt{\left(1 - r_{21}/r_{21}^{\text{res}}\right)^2 + 4m_{2D}^2/m_{3D}^2}}. \tag{5}$$

The other two RH mixing angles are both small and can be expressed in β_{23} as follows

$$\beta_{12} \approx \frac{1}{\sqrt{2}\sin\theta_{12}^{\nu}} \left(-\frac{m_{1D}}{m_{2D}} \cos\beta_{23} + \frac{m_{1D}}{m_{3D}} \sin\beta_{23}\right), \tag{6}$$

$$\beta_{13} \approx \frac{1}{\sqrt{2}\sin\theta_{12}^{\nu}} \left(-\frac{m_{1D}}{m_{2D}} \cos\beta_{23} + \frac{m_{1D}}{m_{3D}} \sin\beta_{23}\right). \tag{7}$$

The behavior of $\sin^2 2\beta_{23}$ (see in Fig. 1) as a function of r_{21} is clearly that of a resonance peaked at $r_{21} = r_{21}^{\text{res}}$. The phenomenon is very like that in the matter-enhanced $\nu_e \leftrightarrow \nu_\mu$

oscillation in the sun except that, in our case, r_{21} plays a part of the effective potential $V = 2\sqrt{2}G_f N_e E_\nu$. Here G_f is the Fermi constant, N_e is the electron number density of the matter and E_ν is the neutrino energy. We can define the relative resonance width δ as that of $r_{21}/r_{21}^{\rm res}$ around $r_{21}^{\rm res}$ for which $\sin^2 2\beta_{23}$ becomes $\frac{1}{2}$ instead of the maximum value, unity. It is given by

$$\delta \approx 4\frac{m_{2D}}{m_{3D}} \tag{8}$$

which is far less than unit. If the two heavier RH Majorana neutrino are degenerate and then the corresponding RH mixing angle $\beta_{23} \approx \pi/4$, one can determine the electron neutrino mass

$$m_1 \approx 1.6 \times 10^{-10}\ \text{eV}. \tag{9}$$

Around this point, $M_1 \approx 1.4 \times 10^7$ GeV and

$$M_2 \approx M_3 \approx 2.6 \times 10^{15}\ \text{GeV} \tag{10}$$

M_2 (M_3) remains at 2.6×10^{15} GeV and M_3 (M_2) increases almost linearly with the decrease of m_1 for $m_1 < (>) 1.6 \times 10^{-10}$ eV and so we can also infer the scale of m_1 from the knowledge of the spectrum of the RH Majorana neutrinos. The above values are obtained by taking $m_2 \approx \sqrt{\Delta m^2_{\rm solar}} \approx \sqrt{5.4 \times 10^{-6}}$ eV and $\sin^2 2\theta^\nu_{12} \approx \sin^2 2\theta_{\rm solar} \approx 6.0 \times 10^{-3}$ [13]. We always take $m_3 \approx \sqrt{\Delta m^2_{\rm atm.}} \approx \sqrt{5.9 \times 10^{-3}}$ eV [2] and $m_D^{\rm diag} \approx m^{\rm up}(\mu)$ [15]. Here $\mu = 10^9$ GeV.

If θ^ν_{12} is large, we find, when $\sin^2\theta^\nu_{13} \ll \frac{1}{2}\frac{m^2_{1D}}{m^2_{2D}}$, the RH Majorana masses are given by

$$M_1 \approx \begin{cases} \frac{m^2_{1D}}{m_2}\frac{1}{\sin^2\theta^\nu_{12}} & \frac{m_3}{m_2} < 2\frac{m^2_{2D}}{m^2_{1D}}\sin^2\theta^\nu_{12} \\ 2\frac{m^2_{2D}}{m_3} & \frac{m_3}{m_2} > 2\frac{m^2_{2D}}{m^2_{1D}}\sin^2\theta^\nu_{12} \end{cases} \tag{11a}$$

$$M_2 \approx \begin{cases} 2\frac{m^2_{2D}}{m_3} & \frac{m_3}{m_2} < 2\frac{m^2_{2D}}{m^2_{1D}}\sin^2\theta^\nu_{12} \\ \frac{m^2_{1D}}{m_2}\frac{1}{\sin^2\theta^\nu_{12}} & \frac{m_3}{m_2} > 2\frac{m^2_{2D}}{m^2_{1D}}\sin^2\theta^\nu_{12} \end{cases} \tag{11b}$$

$$M_3 \approx \frac{1}{2}\frac{m^2_{3D}}{m_1}\sin^2\theta^\nu_{12} \tag{11c}$$

and the RH mixing angles $\beta_{13} \approx \frac{\sqrt{2}m_{1D}}{m_{3D}}\cot\theta^\nu_{12}$ and $\beta_{23} \approx -\frac{m_{2D}}{m_{3D}}$. We have deduce simple expression for β_{13}. In Fig. 2 M_1, M_2 and $\sin^2 2\beta_{12}$ near $r_{32}^{\rm res} \approx 2\frac{m^2_{2D}}{m^2_{1D}}\sin^2\theta^\nu_{12} \sim 10^4$ are numerically plotted taking $m_3^2 = 0.1$ eV^2. The behavior $\sin^2 2\beta_{12}$ as a functions of r_{32} is also a resonance peaked at $r_{32}^{\rm res}$. Taking 10^{-1} eV$^2 < m_3^2 \approx \Delta m^2_{\rm atm.} < 10^{-3}$ eV^2 in consider, the

domain of m_2^2 corresponding to $r_{32}^{\rm res}$ is $10^{-11} - 10^{-13}$ which lies in the region of the vacuum oscillation (VO) solution to the solar neutrino problem. For the large mixing MSW effect and low mass MSW effect, one always has $\frac{m_3}{m_2} < 2\frac{m_{2D}^2}{m_{1D}^2}\sin^2\theta_{12}^\nu$ and so

$$M_1 \approx \frac{m_{1D}^2}{m_2}\frac{1}{\sin^2\theta_{12}^\nu}, \quad M_2 \approx 2\frac{m_{2D}^2}{m_3}, \quad M_3 \approx \frac{1}{2}\frac{m_{3D}^2}{m_1}\sin^2\theta_{12}^\nu; \tag{12}$$

$$\beta_{12} \approx -\frac{1}{\sqrt{2}}\frac{m_{1D}}{m_{2D}}\cot\theta_{12}^\nu, \quad \beta_{13} \approx \sqrt{2}\frac{m_{1D}}{m_{3D}}\cot\theta_{12}^\nu, \quad \beta_{23} \approx -\frac{m_{2D}}{m_{3D}}. \tag{13}$$

For VO, we have the similar tendency of $M_{1,2}$ with the increase of r_{32} as that of $M_{2,3}$ with the increase of r_{21} in the SMA case. So the allowed region of the mass-squared difference Δm_{23}^2 could be confined in a relatively narrow range from the RH Majorana neutrino spectrum. Inserting $m_2 \approx \sqrt{\Delta m_{solar}^2} \approx \sqrt{6.5\times 10^{-11}}$ eV and $\sin^2 2\theta_{12}^\nu \approx \sin^2 2\theta_{solar} \approx 0.75$ [13] in Eq. (11), we obtain $M_1 \approx M_2 \approx 3.6\times 10^9$ GeV and $M_3 \approx 1.8\times 10^{17} r_{21}$ GeV when $r_{32} \approx 4.1\times 10^4$.

To summarize, from the mass spectrum of RH Majorana neutrino, we can infer whether the long wave-length vacuum oscillation or the matter-enhanced conversion is responsible for the solar neutrino deficit. If the small mixing MSW effect is just the case, the electron neutrino mass may be inferred from the structure of mass spectrum of the RH Majorana neutrino. On the other hand, if the solar neutrino deficit is due to the vacuum oscillation, the structure of such a spectrum could constrain the region of the $m_{2,3}$. However, $M_3(\gg 1.8\times 10^{17}$ eV) is too large to be believable in VO case. Note that $M_{2,3}$ are near the unification scale ($\sim 10^{16}$ GeV) [7] in the supersymmetric case while M_1 is even far less than the intermediate scale ($\sim 10^9 - 10^{13}$ GeV) [7,16]. It would be interesting if the information about the spectrum of the RH Majorana neutrino can be corroborated by other theories such as the supersymmetry theory or the string.

The results are dependent on the precise determination of U_{e3}. Such a goal is expected to be reached in the undergoing or forthcoming neutrino long baseline experiment, registration of the neutrino bursts from the Galactic supernova by existing detectors SK and SNO, and the neutrino factories [17].

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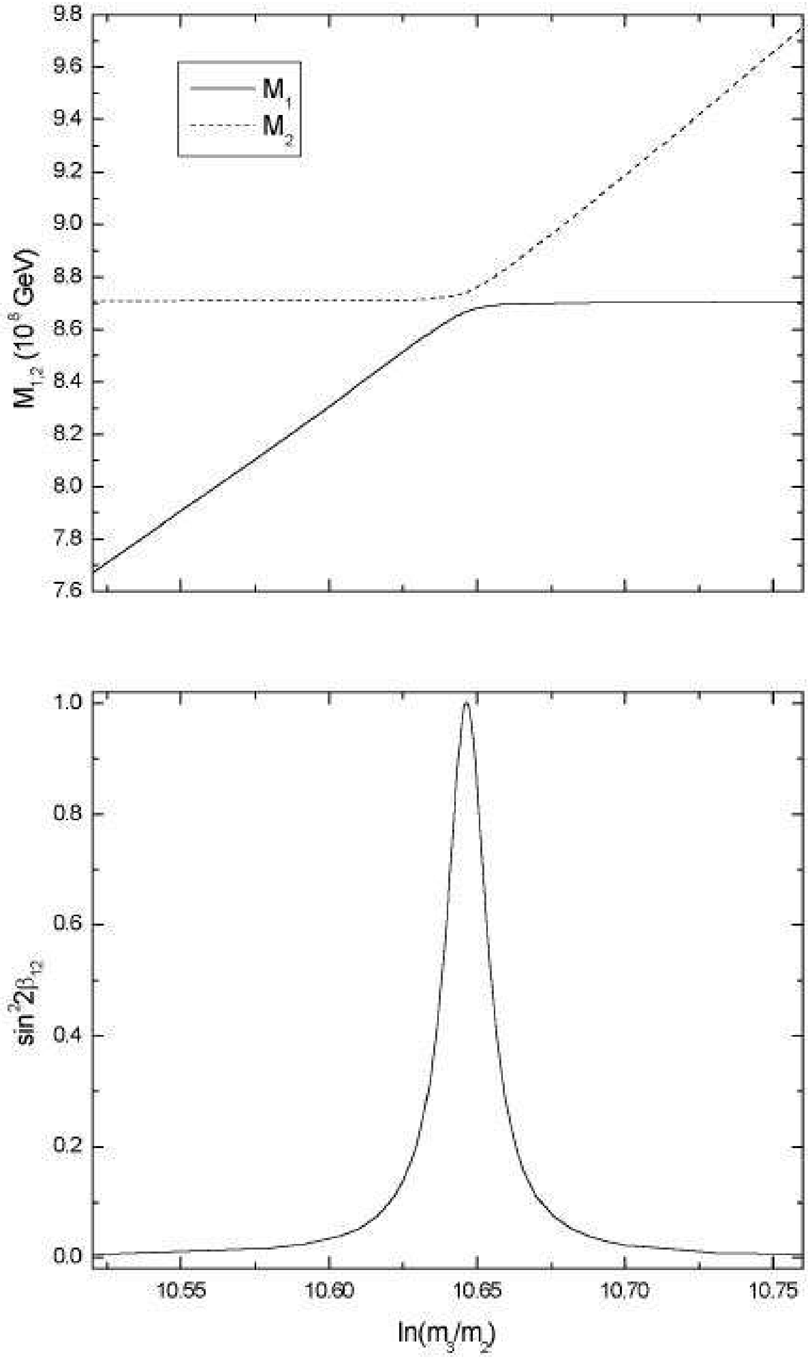
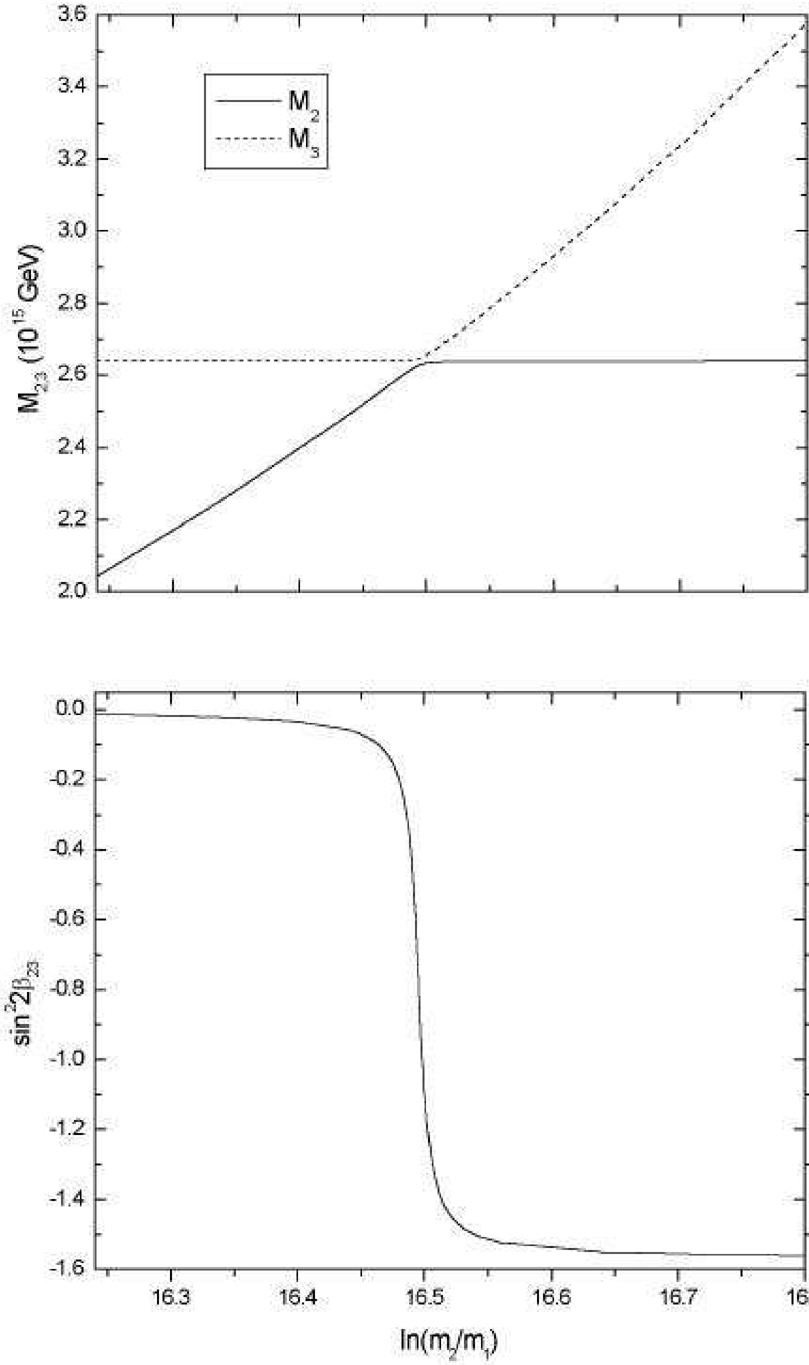
FIGURES

FIG. 1. The behavior of the M_2, M_3 and β_{23} as a function of $\frac{m_2}{m_1}$ for the SMA solution to the solar neutrino problem taking $U_{\tau 1} = 0$. We take $m_2^2 = 5.4 \times 10^{-6}$ eV^2, $m_3^2 = 5.9 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{12}^{\nu} = 6.0 \times 10^{-3}$, $\theta_{13}^{\nu} = 0.0$ and $\sin^2 2\theta_{23}^{\nu} = 1.0$.

FIG. 2. The behavior of M_1, M_2 and $\sin^2 2\beta_{12}$ as a function of $\frac{m_3}{m_2}$ for the vacuum oscillation solution to the solar neutrino problem taking $U_{e3} = 0$. $\sin^2 2\theta_{12}^{\nu} = 0.75$ and $\sin^2 2\theta_{23}^{\nu} = 1.0$.



M1
M2
M1,2 (10^8 GeV)
sin^2 2β12
ln(m3/m2)



M_2
M_3
M_{2,3} (10^15 GeV)
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
sin^2 2β_23
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
16.3
16.4
16.5
16.6
16.7
16.8
ln(m_2/m_1)